

Shanghai University of Finance and Economics

Entrance Examination for 2024 Interdisciplinary Sciences Elite Program

Date: September 06, 2024

Time: 12:00pm – 3:00pm

There are 7 questions, and total score is 100.

1 Background

Mobile network providers typically offer their customers a variety of different data plans. In this problem, we examine how different pricing methods affect the profit (利润) in various scenarios.

Consider the following two pricing methods: volume-based pricing (按量收费) and monthly subscription fee (包月收费). In the first method, the plan has a single parameter R (RMB/GB), meaning that the customer will be charged $R \times D$ RMB if D GB are used. In the second method, the plan has two parameters F (RMB/month) and C (GB), meaning that the customer will be charged F RMB each month as long as no more than C GB are used.

We assume the cost (成本) for providing data is negligible (可忽略), and so the profit of the company is simply the total amount of money paid by all customers.

2 Customers with fixed demands and fixed budgets

In a market survey, we observe that there are N customers of the same type. They all have the same monthly demands (需求) D GB and are willing to pay up to P RMB for this service. Note that the customers will not pay anything if their demands cannot be fulfilled within their budgets (预算).

Question 1 (10 pts): What are the maximum monthly profits for each of the pricing methods, and the corresponding parameters R , F , C ? Which pricing method is better for the company?

Answer 1: The maximum profits are $N \times P$ RMB in both cases. The profit cannot be higher as each customer are willing to pay at most P RMB and there are N of them.

For volume-based pricing, optimal $R = P/D$, so each customer pays P RMB.

For monthly subscription, optimal $F = P$ and $C \geq D$.

Both pricing methods are equally good.

Suppose there are 2 types of customers instead. There are N low-end customers and N high-end customers. The low-end customers are willing to pay P_1 RMB for D_1 GB/month, and the high-end customers are willing to pay P_2 RMB for D_2 GB/month. We assume that $P_1 < P_2 \leq 2P_1$, $D_1 < D_2$, and $P_1/D_1 < P_2/D_2 \leq 2P_1/D_1$.

Question 2 (10+10 pts): What is the profit using volume-based pricing with parameter R ? You can consider different ranges of R and give a formula for each of the cases.

What is the maximum profit and the corresponding R ?

Answer 2: We consider the following three cases.

Case 1: $0 \leq R \leq P_1/D_1$. The profit is $R \times N \times (D_1 + D_2)$. Maximized at $R = P_1/D_1$.

Case 2: $P_1/D_1 < R \leq P_2/D_2$. The profit is $R \times N \times D_2$. Maximized when $R = P_2/D_2$.

Case 3: $P_2/D_2 < R$. The profit is 0.

So, if $(P_1/D_1) \times N \times (D_1 + D_2) \geq (P_2/D_2) \times N \times D_2 = NP_2$, then the optimal $R = P_1/D_1$ and the profit is $NP_1 + NP_1D_2/D_1$. Indeed, we have $P_1(1 + D_2/D_1) \geq P_1(1 + P_2/(2P_1)) = P_1 + P_2/2 \geq P_2$.

Question 3 (10+5 pts): What is the maximum profit using monthly subscription fee, and the corresponding parameters F , C ?

Compared with the volume-based pricing in Question 2, which pricing method is better for the company?

Answer 3: If we want to fulfill high-end customers only, the optimal (F, C) can be chosen to be (P_2, D_2) as the most we can get from each of them is P_2 . The corresponding profit is NP_2 .

If we want to fulfill all customers, the optimal (F, C) can be chosen to be (P_1, D_2) and the corresponding profit is $2NP_1$.

Since $2P_1 \geq P_2$, the optimal $(F, C) = (P_1, D_2)$ and the maximum profit is $2NP_1$.

Since $D_2/D_1 > 1$, $NP_1 + NP_1D_2/D_1 > 2NP_1$ and hence volume-based pricing is better than monthly subscription.

3 Customers with diminishing marginal utilities

In reality, customers often value (估值) the first few GBs higher than the rest, which means they are willing to pay more for the 1st GB than the 100th GB. This effect is called the law of diminishing marginal utilities (边际效用递减).

We assume that a customer with maximum demand D GB is willing to pay D RMB for the 1st GB, $D - 1$ RMB for the 2nd GB, $D - 2$ RMB for the 3rd and so on, until 1 RMB for the n -th GB. The total amount is thus the area of Figure 1. For simplicity, we approximate (近似) the figure with a triangle, and hence the customer is willing to pay the area of the trapezoid $Dx - x^2/2$ RMB for x GBs when $0 \leq x \leq D$, and $D^2/2$ RMB for more than D GB. Formally, when a customer uses x GBs, we define his valuation function (估值函数) as

$$V(x) = \begin{cases} Dx - x^2/2 & \text{if } 0 \leq x \leq D, \\ D^2/2 & \text{if } x > D. \end{cases}$$

Note that now x can also be fractional (小数).

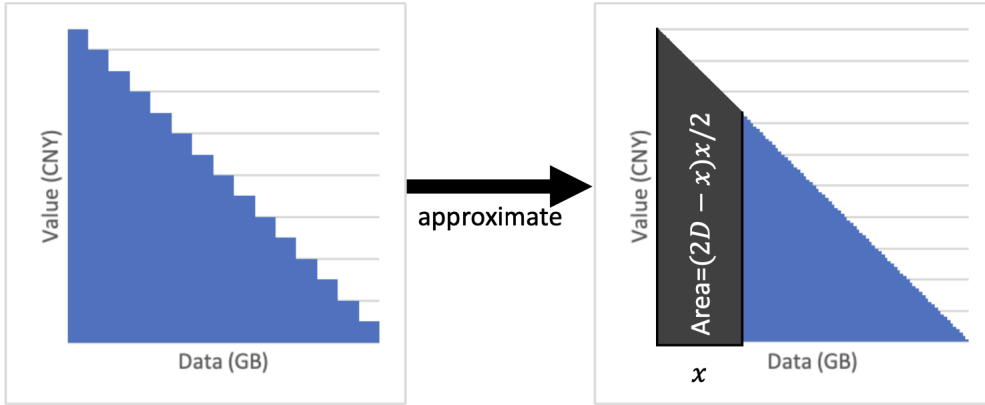


Figure 1: The customer values the first x GB at $(2D - x)x/2$ RMB.

The customer's utility function (效用函数) is defined as his valuation function subtracted by the amount he actually pays. In this scenario, the customer always chooses the x that maximizes (最大化) his utility function. In case of tie (相同效用), we assume the customer is willing to pay the most.

Suppose there are N customers with maximum demands D .

Question 4 (10+5 pts): What is the profit using volume-based pricing with parameter R ? Note that the customers may choose to use less than D GB because of the diminishing utility.

What is the maximum profit and the corresponding R ?

Answer 4: The utility of each customer for using x GB is $Dx - x^2/2 - Rx$ for $0 \leq x \leq D$ and $0 \leq R \leq D$, and it maximizes when $x = D - R$. Hence the profit is $R(D - R)N$. When $R > D$ the profit is 0 as the customers have negative utility for using any data.

The maximum profit is $D^2N/4$ at $R = D/2$.

Question 5 (5+5 pts): What is the maximum profit using monthly subscription fee, and the corresponding parameters F , C ? Compared with the volume-based pricing in Question 4, which pricing method is better for the company in this scenario?

Answer 5: The maximum profit is $D^2N/2$ by setting $(F, C) = (D^2/2, D)$. The customers have 0 utility using this plan and negative utility for any plan with higher subscription fee.

The monthly subscription is always better in this scenario.

Suppose there are N customers with maximum demands D_1 and N customers with maximum demands D_2 . We assume that $D_1 < D_2 \leq 2D_1$.

Question 6 (10+5 pts): What is the profit using volume-based pricing with parameter R ?

What is the maximum profit and the corresponding R ?

Answer 6: For $0 \leq R \leq D_1$, the profit is $R \times (D_1 - R) \times N + R \times (D_2 - R) \times N$. For $D_1 < R \leq D_2$, the profit is $R \times (D_2 - R) \times N$. For $D_2 < R$, the profit is 0.

Since the second case maximizes at $R = D_2/2$ which is $\leq D_1$ by assumption, we conclude that cases 2 and 3 are never optimal.

The function $RN(D_1 - R + D_2 - R)$ maximizes at $R = (D_1 + D_2)/4$ and the maximum profit is $(D_1 + D_2)^2N/8$.

Question 7 (10+5 pts): What is the maximum profit using monthly subscription fee, and the corresponding parameters F , C ? Compared with the volume-based pricing in Question 6, which pricing method is better for the company in this scenario?

Answer 7: If we want to fulfill high-end customers only, the optimal (F, C) can be chosen to be $(D_2^2/2, D_2)$ and the corresponding profit is $ND_2^2/2$.

If we want to fulfill all customers, the optimal (F, C) can be chosen to be $(D_1^2/2, D_2)$ and the corresponding profit is ND_1^2 .

So, the maximum profit is $\max(ND_2^2/2, ND_1^2)$, with corresponding parameters $(D_2^2/2, D_2)$ and $(D_1^2/2, D_2)$. Since $\max(ND_2^2/2, ND_1^2) \geq ND_2^2/2 \geq N(D_1 + D_2)^2/8$ as $D_2 > D_1$, we conclude that monthly subscription is always better in this scenario.